[1]J. V. Allaby et al., Phys. Letters 28B, 67 (1968).
[2]M. N. Kreisler, F. Martin, M. L. Perl, M. J. Longo, and S. T. Powell, III, Phys. Rev. Letters 16, 1217 (1966).
[3]J. V. Allaby et al., to be published.
[4]G. Cocconi et al., Phys. Rev. 126, 277 (1962).
[5]F. Bradamante et al., unpublished.
[6]R. Karplus and Y. Yamaguchi, Nuovo Cimento 22, 588 (1961).
[7]R. J. Glauber, in Lectures in Theoretical Physics, edited by W. E. Brittin et al. (Interscience, New York, 1959), Vol. 1.
[8]L. Bertocchi, Nuovo Cimento 50A, 1015 (1967).
[9]M. Verde, Helv. Phys. Acta 22, 339 (1949).
[10]The integrated spectrum can be very easily calculated, but it gives no information about the overlap of the two peaks in Fig. 1.
[11]A. D. Krisch, Phys. Rev. 135, 1456 (1964).
[12]The interference between single scattering off the proton and neutron is negligible away from the forward direction.
[13]E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. 139, B458 (1965).
[14]F. Gross, in Medium Energy Nuclear Physics, edited by W. Bertozzi and S. Kowalski (Massachusetts Institute of Technology, Cambridge, Mass., 1967).
[15]I. J. McGee, Phys. Rev. 161, 1640 (1967).
[16]G. Belletini et al., Phys. Letters 18, 167 (1965).

PHOTON CROSS SECTIONS AND VECTOR DOMINANCE*

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(Received 19 December 1969)

We calculate (1) incoherent photoproduction of ρ^0 mesons, (2) total photon-nucleus cross sections, and (3) incoherent photoproduction of positive pions within the framework of simple vector dominance. Comparison is made with experiment.

The multi-GeV elastic scattering of photons by nuclei and the incoherent photoproduction of ρ^0 and π mesons provide interesting tests of vector dominance. There have been calculations of these processes using eikonal methods which will be referred to below, and now a considerable amount of experimental data has become available. Although there are no photon elastic-scattering results from nuclei, there are total photon cross-section measurements, which can be simply related to the forward-scattering amplitude through the optical theorem.

The object of this Letter is to make calculations of these processes in some detail, assuming simple vector-meson dominance. We will need vector-dominance coupling constants, certain scattering and production amplitudes on nucleons, and nuclear size parameters. All these we take from other experiments. Our calculations have no free parameters then.

Incoherent photoproduction of ρ^0 mesons.—In a recent Letter[1] we have calculated the incoherent production of ρ^0 mesons on nuclei under the assumption of vector dominance using eikonal methods.[2,3] The process is calculated as a combination of (a) a one-step process corresponding to diffractive photoproduction of a ρ^0 meson on a nucleon accompanied by nuclear excitation, and (b) coherent photoproduction of a ρ^0 meson on a nucleon (no nuclear excitation) followed by incoherent scattering of the ρ^0 meson (nuclear excitation occurs). Appropriate diagrams are shown in Fig. 1(a). Since the detailed formulas have been written down before,[1,3,4] we only note here that we can write the incoherent cross section in the form

$$d\sigma^{(1)}(\gamma A \to \rho^0 A')/d\Omega = |f_{\gamma\rho}(t)|^2 N_{\rm eff}, \tag{1}$$

where $N_{\rm eff}$ is an effective nucleon number and $f_{\gamma\rho}(t)$ is the two-body photoproduction amplitude, assumed spin and isospin independent.

We have redone the calculation for incoherent ρ^0 photoproduction allowing for a nonzero real part for the ρ^0-nucleon forward-scattering amplitude, taken from pion-nucleon scattering[5] using the quark-model relation

$$f_{\rho^0 p}(0) = \tfrac{1}{2}[f_{\pi^+ p}(0) + f_{\pi^- p}(0)]$$

to determine the real part of $f_{\rho^0 N}(0)$. The total cross section for a ρ^0 meson on a nucleon $\sigma_{\rho N}$, related by the optical theorem to the imaginary part of the forward-scattering amplitude $f_{\rho^0 N}(0)$, has been determined using the vector-dominance relation[6]

$$\left.\frac{d\sigma_{\gamma\rho}}{dt}\right|_{t=0} = \frac{1}{16}\frac{\alpha}{4\pi}\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1}\sigma_{\rho N}^2(1+\beta^2), \tag{2}$$

where $\beta = \mathrm{Re}\, f_{\rho^0 N}(0)/\mathrm{Im}\, f_{\rho^0 N}(0)$ and $\gamma_\rho^2/4\pi = 0.5$, taken from the compilation of Ting.[6] The values of $d\sigma_{\gamma\rho}/dt$ are taken from a fit to data of ρ^0 photoproduction on hydrogen.[7] These values and other parameters for the calculation are listed in

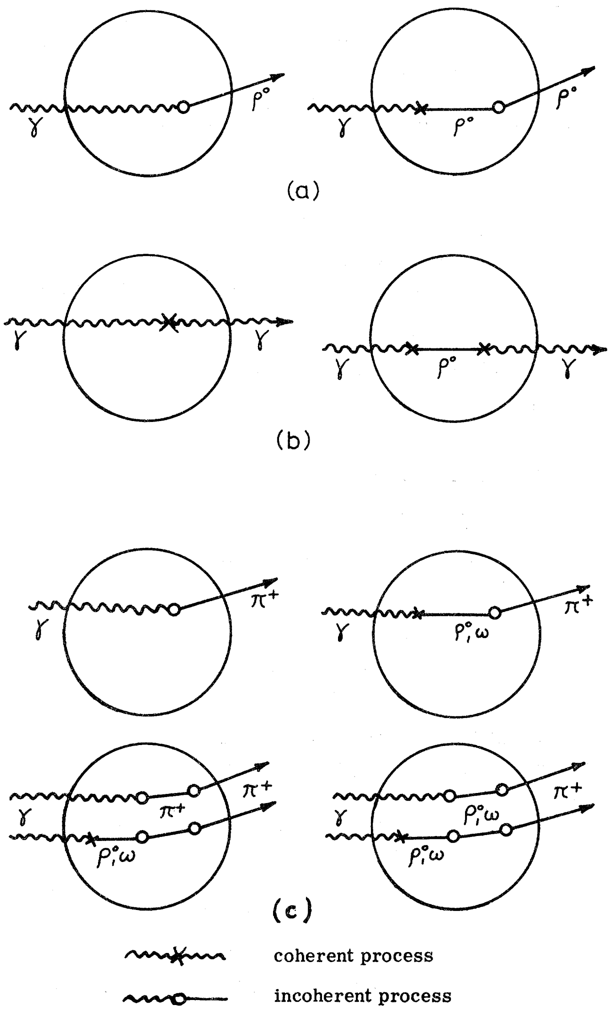

FIG. 1. Diagrams for one-step and multistep photoreaction processes. (a) Incoherent ρ^0 production. (b) Photon forward elastic scattering. (c) Incoherent π^+ photoproduction.

Table I. We use the average nuclear radius $a = 1.12A^{1/3}$ fm obtained from coherent ρ^0 photoproduction,[8] the nucleus being assumed to have a Woods-Saxon shape with thickness parameter $c = 0.545$ fm. In the calculations described here we ignore the effects of nuclear correlations. For both coherent and incoherent processes, correlations lead to a small renormalization of the two-body total cross sections involved.[9] For incoherent processes there is, in addition, a new t dependence due to the Fourier transform of the two-body correlation function.[9] We comment on this again further on in the paper. The results for N_{eff} as a function of A and of photon lab energy are shown in Fig. 2. The differences between this calculation and that of Ref. 1 are small, and we conclude, as in Ref. 1, that there is no apparent disagreement with experiment.[10]

Photon-nucleus total cross sections.—Total photon-nucleus cross sections have been calculated by several groups.[11] There are now experimental data[12] confirming the presence of a nuclear shadow.[11,13] One calculates the forward photon-nucleus scattering amplitude using eikonal methods and then invokes the optical theorem to get the total cross section. The scattering amplitude, according to vector dominance, is the sum of two amplitudes [see Fig. 1(b)], the diffractive scattered amplitude corresponding to photon scattering on each nucleon, summed over all nucleons (proportional to nucleon number A), and a two-step regenerative amplitude corresponding to the photoproduction of one of the vector mesons ρ, ω, or φ on one nucleon, followed by radiative capture on another nucleon. The coupling between the vector mesons ω and φ is neglected. The damping of the intermediate vector mesons by the target nucleons creates the shadow.

The results of our calculation for the photon-nucleus total cross section and data from Ref. 12 are shown in Fig. 3. As in incoherent ρ^0 photoproduction, we note a comparatively weak energy dependence of the photon total cross sections. This is due to the decrease of $\sigma_{\rho N}$ with energy and to the presence of a nonzero real part in the ρ-nucleon forward-scattering ampli-

Table I. The parameters for the calculation as functions of the incident photon energy.

E_γ (GeV)	$[d\sigma(\gamma p\to\rho^0 p)/dt]_{t=0}$ (fit of exptl data) (μb/GeV^2)	$\beta = \mathrm{Re}f/\mathrm{Im}f$	$\sigma_{\pi^+}{}^{\mathrm{tot}}$ (mb)	$\sigma_{\rho p}{}^{\mathrm{tot}}$ from VMD[a] with $\gamma^2/4\pi = 0.5$ (mb)
3	152	-0.26	···	27.6
5	124.2	-0.22	···	25.2
8	113.1	-0.185	25	24.2
16	106.2	-0.135	24	23.65

[a]Vector-meson dominance.

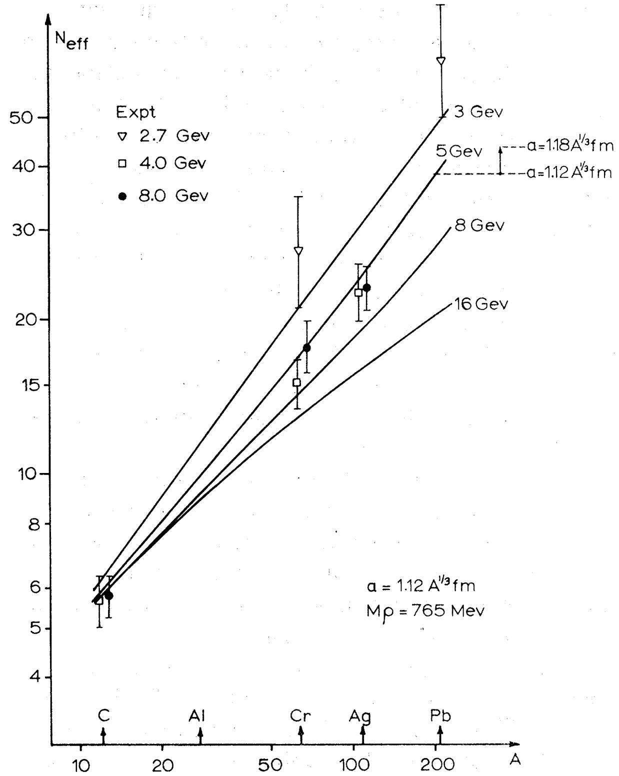

FIG. 2. ρ^0 photoproduction: Experimental and theoretical values of N_{eff} for several energies as a function of A. The effect of a change in the nuclear radius from $a=1.12A^{1/3}$ fm to $a=1.18A^{1/3}$ fm is shown for $A=208$ at $E_\gamma=5$ GeV.

tude. It may further be noted that the nuclear shadow is not strong; $\sigma(\gamma A)\propto A^{0.9}$ for $k\simeq 8$ GeV/c. This reflects the long mean free path of ρ^0 mesons in the nucleus; $\sigma_{\rho N}\simeq 25$ mb corresponds to a mean free path of over 3 fm.

Incoherent production of π^+ mesons.—Lastly we calculate the incoherent photoproduction of π^+ mesons in nuclei. We include the processes of Fig. 1(c). The first two diagrams of Fig. 1(c) correspond to only one incoherent step. Their contribution, similar to the calculation of incoherent ρ^0 production (see Gottfried and Yennie, Ref. 3), can be written in the form

$$\frac{d\sigma}{dt}(\gamma A\to\pi^+A')=\frac{Z}{A}\frac{d\sigma}{dt}(\gamma p\to\pi^+n)N_{\mathrm{eff}},\tag{3}$$

where N_{eff} is an effective nucleon number which at infinite energy becomes

$$N_{\mathrm{eff}}=N(A;\sigma_\rho,\sigma_{\pi^+})=(\sigma_\pi-\sigma_\rho)^{-1}\int[e^{-\sigma_\rho T(b)}-e^{-\sigma_\pi T(b)}]d^2b.\tag{4}$$

Here $T(b)=A\int_{-\infty}^{\infty}\rho(b,z)dz$ where $\rho(b,z)$ is the single-particle nucleon density function. The incom-

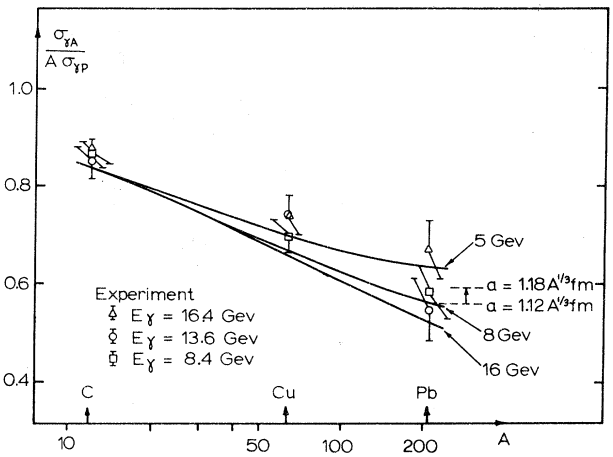

FIG. 3. Total photon cross-section calculations and data from Ref. 12. For the ω and φ meson contribution the following parameters are used: $\sigma_\omega=\sigma_\rho$, $\sigma_\varphi=12$ mb, $\gamma_\omega^2/4\pi=4.69$, $\gamma_\varphi^2/4\pi=3.04$. We take $m_\rho=765$ MeV, $m_\omega=783$ MeV, $m_\varphi=1.02$ BeV and the same phase $\beta=\mathrm{Re}f/\mathrm{Im}f$ from the table for all amplitudes. The effect of a change in the nuclear radius from $a=1.12A^{1/3}$ fm to $a=1.18A^{1/3}$ fm is shown for $A=208$ at $E_\gamma=8$ GeV.

ing photon behaves as though it were a ρ^0 meson here.

The last two diagrams of Fig. 1(c) correspond to two incoherent steps. Appropriate formulas have been derived[14] in the literature. We note that, with the assumption that the elastic scattering amplitudes $f_{\pi^+N}=f_{\rho^0N}$, one has for the two incoherent step contributions in the limit of high energy, where the incoming photon acts like a vector meson,

$$\frac{d\sigma^{(\mathrm{II})}}{dt}(t)=\frac{d\sigma_0}{dt}(t)[2N_2(A;\sigma)]\frac{\sigma_{\mathrm{el}}}{\sigma}\frac{a}{a+a_{12}}\times\exp\left\{\frac{a_{12}^2}{a_{12}+a}|t|\right\};\tag{5}$$

and in the low-energy limit ($R_{\mathrm{nucl}}m^2\rho/2k\gg1$),

$$\frac{d\sigma^{(\mathrm{II})}}{dt}(t)=\frac{d\sigma_0}{dt}(t)[N(A;0,\sigma)-N_1(A,\sigma)]\times\frac{\sigma_{\mathrm{el}}}{\sigma}\frac{a}{a+a_{12}}\exp\left\{\frac{a_{12}^2}{a_{12}+a}|t|\right\},\tag{6}$$

where

$$N_m(A,\sigma)=(1/m!)\int d^2b\sigma^{m-1}[T(b)]^m e^{-\sigma T(b)}.\tag{7}$$

Here σ_{el} (σ) is the elastic (total) cross section of a pion or ρ^0 meson on a nucleon, and a (a_{12}) is the slope of the elastic (production) differential cross section of the pion or ρ^0 meson, respectively.

For intermediate energies we evaluate the two-

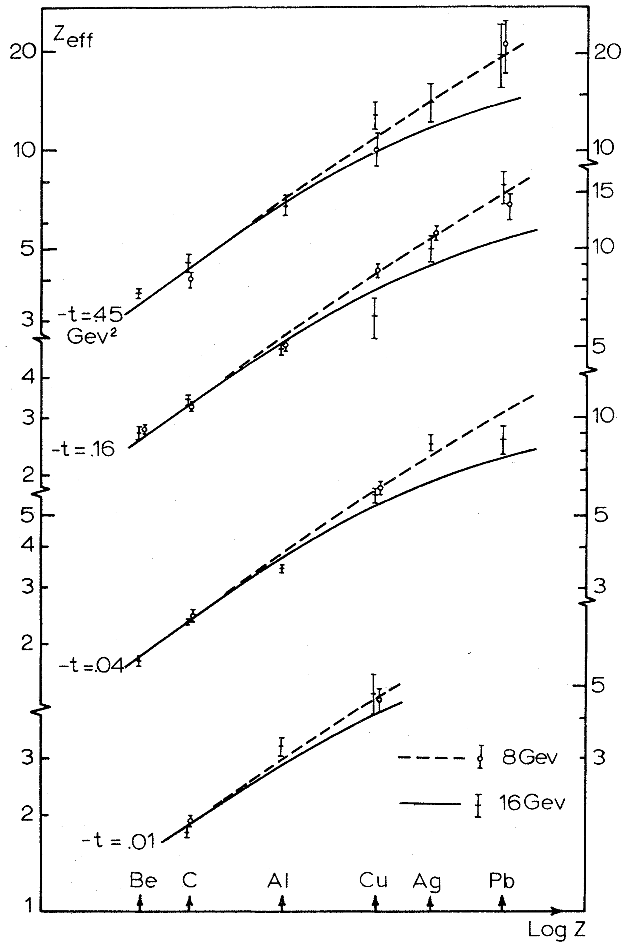

FIG. 4. The A dependence of

$$Z_{\text{eff}}=\frac{d\sigma(\gamma A\rightarrow\pi^+A')/dt}{d\sigma(\gamma p\rightarrow\pi^+n)/dt}$$

for four different momentum transfers. The errors of the data from Ref. 15 are statistical only.

step contribution to the cross section approximately by linear interpolation between these two formulas with the parameter

$$x=\frac{[N_{\text{eff}}-N_1(A,\sigma)]}{[N(A;0,\sigma)-N_1(A;\sigma)]};$$

where $x=0$ ($x=1$) corresponds to the high (low) energy limit.

Figure 4 shows a comparison of calculated $Z_{\text{eff}}=(Z/A)N_{\text{eff}}$ (including two incoherent steps) with π^+ photoproduction measurements.[15] The normalization of the calculated cross section for every value of momentum transfer is fitted to the experimental data. This removes any strong need to calculate the momentum-transfer-dependent incoherent correlation effects.[9] It would be of interest to study these effects, of course, but we leave this for later study. The contribution of two incoherent steps could have been included in the calculation of incoherent ρ^0 production as well. They are of the order of 10 to 20% for $|t|\simeq 0.1$ (GeV/c)^2 (the momentum transfer region of the measurements) in this case. We do not give detailed results here. It is to be noted that the effects of correlations[9] introduce changes of the same order of magnitude in the opposite direction for the ρ^0 photoproduction data.

Is vector dominance satisfied?—We conclude from the above that vector dominance,[16] together with the eikonal methods used, provides a reasonable description of the interesting measurements. Experiment and theory agree to within 10%. The photon total-cross-section measurements lie slightly above the calculated values and it would be interesting to try to decrease the experimental uncertainty. Further measurements on incoherent production of ρ^0 mesons and pions would also be of interest.

We wish to thank Professor D. O. Caldwell, Professor R. Diebold, Professor G. E. Fischer, and Professor S. C. C. Ting for presenting us with the results of their interesting experiments before publication. We also thank Professor D. G. Ryan for providing us with experimental information on the real part of the pion-nucleon scattering amplitude, and Professor J. Charap for interesting discussions.

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*Work supported in part by the National Research Council of Canada.

[1]G. von Bochmann and B. Margolis, Phys. Rev. Letters 23, 935 (1969).

[2]R. J. Glauber, in Lectures in Theoretical Physics, edited by W. E. Brittin et al. (Interscience, New York, 1959); G. von Bochmann and B. Margolis, "Multi-Step Contributions to Particle Production in Nuclei" (to be published).

[3]M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966); S. D. Drell and J. S. Trefil, Phys. Rev. Letters 16, 552, 832 (1966); K. Gottfried and D. R. Yennie, Phys. Rev. 182, 1595 (1969).

[4]B. Margolis, in Proceedings of the International Conference on High Energy Physics and Nuclear Structure, Columbia University, New York, 8-13 September 1969 (to be published).

[5]K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters 19, 193 (1967); H. I. Saxer, University of Michigan Technical Report No. 03106-19-T, 1964 (unpublished).

[6]S. C. C. Ting, in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September 1968, edited by J. Prentki

and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 43.

[7]H. Alvensleben, U. Becker, W. K. Bertram, M. Chen, K. J. Cohen, T. M. Knasel, R. Marshall, D. J. Quinn, G. H. Sanders, H. Schubel, and S. C. C. Ting, DESY Report No. 69/32, 1969 (to be published); R. Anderson, D. Gustavson, J. Johnson, D. Ritson, B. H. Wiik, W. G. Jones, D. Kreinick, F. Murphy, and R. Weinstein, Stanford Linear Accelerator Center Report No. SLAC-PUB-644, 1969 (to be published), as quoted in D. O. Caldwell, V. B. Elings, W. P. Hesse, G. E. Jahn, R. J. Morrison, F. V. Murphy, and D. E. Yount, Phys. Rev. Letters 23, 1256 (1969).

[8]H. Alvensleben, U. Becker, W. K. Bertram, M. Chen, T. M. Knasel, R. Marshall, D. J. Quinn, M. Rohde, G. H. Sanders, H. Schubel, and S. C. C. Ting, "On the Photo-production of Neutral ρ-Mesons from Complex Nuclei" (to be published).

[9]G. von Bochmann, B. Margolis, and C. L. Tang, Phys. Letters 30B, 254 (1969).

[10]J. G. Asbury, U. Becker, W. K. Bertram, P. Joos, M. Rohde, A. J. S. Smith, C. L. Jordan, and S. C. C. Ting, Phys. Rev. Letters 19, 865 (1967); G. McClellan, N. Mistry, P. Mostek, H. Ogren, A. Osborne, A. Silverman, J. Swartz, R. Talman, and G. Diambrini-Palazzi, Phys. Rev. Letters 23, 554 (1969).

[11]M. Nauenberg, Phys. Rev. Letters 22, 556 (1969); Gottfried and Yennie, Ref. 3; S. J. Brodsky and J. Pumplin, Phys. Rev. 182, 1794 (1969); B. Margolis and C. L. Tang, Nucl. Phys. B10, 329 (1969).

[12]Caldwell et al., Ref. 7.

[13]J. S. Bell, Phys. Rev. Letters 18, 57 (1964); CERN Report No. TH. 877 (unpublished).

[14]K. S. Kölbig and B. Margolis, Nucl. Phys. B6, 85 (1968); R. J. Glauber and G. Matthiae, Istituto Superiore di Sanitâ, Laboratōri di Fisica, Roma, Report No. ISS 67/16, 1967 (unpublished).

[15]A. M. Boyarski, R. Diebold, S. D. Ecklund, G. E. Fischer, Y. Murata, B. Richter, and M. Sands, Phys. Rev. Letters 23, 1343 (1969).

[16]See J. J. Sakurai, in Proceedings of the International Conference on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969 (unpublished).

DUAL PARTNER FOR THE POMERANCHUKON

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We postulate a new Regge singularity which together with the Pomeranchukon forms a "dual set." As a result, there is associated with the Pomeranchukon background of Harari and Freund an additional contribution which occurs, however, only in channels with the quantum numbers of the vacuum. The new singularity $\alpha_{\rm DP}$ couples to the (exotic) $I=2$ $\pi\pi$ channel and probably has $\alpha_{\rm DP}(0)\geqslant 0$. It is also postulated that (other) Regge cuts form a further "dual set," and are therefore built out of backgrounds, not out of resonances; hence the generalized interference model should be valid for cut-dominated processes.

In this note we discuss the idea that the background imaginary parts[1] generated by Pomeranchukon exchange are accompanied by an additional imaginary part of the same order of magnitude, which however occurs only when the direct channel has the quantum numbers of the vacuum. Such a phenomenon occurs in both the models so far proposed to include the Pomeranchukon in $\pi\pi$ scattering, but unfortunately they both have troublesome features.[2] In this note we give a largely model-independent discussion, the important points being the following:

(1) Integrals over the Pomeranchukon contribution and over the extra contribution may be related using exact "crossing sum rules."

(2) The asymptotic behavior of the extra contribution will be controlled by some J-plane singularity whose position we denote by $\alpha_{\rm DP}(t)$ ("direct Pomeranchukon"). We show that the Adler condition for $\pi\pi$ scattering would suggest $\alpha_{\rm DP}(0)\geqslant 0$.

(3) We conjecture that the above "direct Pomeranchukon" singularity and the usual "exchange Pomeranchukon" singularity comprise a "dual set" in a certain sense. Another dual set in this same sense is presumably formed by the usual exchange-degenerate trajectories (ρ, f, etc.).

(4) We further conjecture that Regge cuts (apart from the exchange and direct Pomeranchukons) form a third dual set; it would then follow that Regge cuts are built out of background, not out of resonances.

(1) Recently there has been some interest in "crossing sum rules," which follow when a fixed-t and a fixed-u dispersion relation are required to give the same answer.[3] There are three simple sum rules of this kind for $\pi\pi$ scattering (besides an infinite number of others involving higher derivatives),